Hydraulic losses in pipes

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Contents

1 Viscous flows in pipes

Our intension here is generalized the one-dimensional Bernoulli equation for viscous flow. When the viscosity of the fluid is taken into account total energy head $H = \frac{v^2}{2g} + \frac{p}{\rho g} + z$ is no longer constant along the pipe. In direction of flow, due to friction cause by viscosity of the fluid we have $\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 > \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2$. So to restore the equality we must add some scalar quantity to the right side of this inequality

$$\boxed{\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + \Delta h_{ls}} \tag{1}$$

This scalar quantity Δ_{ls} is called as *hydraulic loss*. The hydraulic loss between two different cross section along the pipe is equal to the difference of total energy for this cross section:

$$\boxed{\Delta h_{ls} = H_1 - H_2} \tag{2}$$

We must remember that always $H_1 > H_2$. In horizontal pipe when $z_1 = z_2$ and diameter of pipe is constant $v_1 = v_2$ hydraulic loss is equal to the head of pressure drop or *head loss*

$$\Delta h_L = \frac{p_1 - p_2}{\rho g} \tag{3}$$

Head loss is express by Darcy -Weisbach equation:

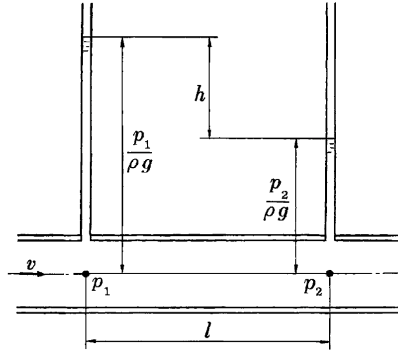$$\boxed{h_L = f \, \frac{L}{D} \, \frac{v^2}{2g}} \tag{4}$$

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Figure 1: Pipe friction loss. For horizontal pipe, with constant diameter this loss may be measured by height of the pressure drop: $\frac{\Delta p}{\rho g} = h$

We must remember that equation (4) is valid only for horizontal pipes. In general, with $v_1 = v_2$ but $z_1 \neq z_2$, the head loss is given

$$\frac{p_1 - p_2}{\rho g} = (z_2 - z_1) + f \frac{L}{D} \frac{v^2}{2g} \qquad (5)$$

Part of the pressure change is due to elevation change and part is due to head loss associated with frictional effects, which are given in terms of the *friction factor* f that depends on Reynolds number and relative roughness $f = \varphi(Re, \varepsilon/D)$.

It is not easy to determine the functional dependence of the friction factor on the Reynolds number and relative roughness (ε/D). Much of this information is a result of experiments conducted by J. Nikuradse in 1933 and amplified by many others since then. One difficulty lies in the determination of the roughness of the pipe. Nikuradse used artificially roughened pipes produced by gluing sand grains of known size onto pipe walls to produce pipes with sandpaper-type surfaces. In commercially available pipes the roughness is not as uniform and well defined as in the artificially roughened pipes used by Nikuradse. However, it is possible to obtain a measure of the effective relative roughness of typical pipes and thus to obtain the friction factor. Figure (3)) shows the functional dependence of f on Re and and is called the **Moody chart** in honor of L. F. Moody, who, along with C. F. Colebrook, correlated the original data of Nikuradse in terms of the relative roughness of commercially available pipe materials.

1.1 Moody Chart

The following characteristics are observed from the data of (3). For laminar flow, $Re < 2300, f = 64/Re$, which is independent of relative roughness. For very large Reynolds numbers, $f = \varphi(\varepsilon/D)$ which is independent of the Reynolds number. For such flows, commonly termed *completely turbulent* flow, along the wall pipe, exists the laminar sublayer so thin that the surface roughness completely dominates the character of the flow near the wall. The gap in the figure for which no values of f are given, $2100 < Re < 4000$, is a result of the fact that the flow in this transition range
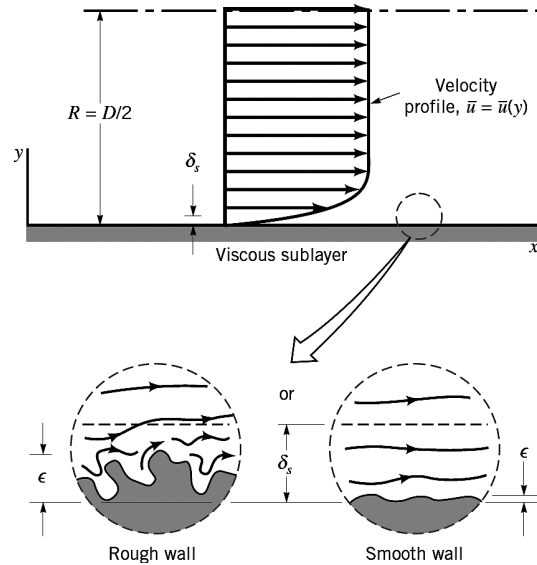
Figure 2: Flow near rough and smooth walls

may be laminar or turbulent (or an unsteady mix of both) depending on the specific circumstances involved.

Note that even for smooth pipes the friction factor is not zero. That is, there is a head loss in any pipe, no matter how smooth the surface is made. This is a result of the no-slip boundary condition that requires any fluid to stick to any solid surface it flows over. There is always some microscopic surface roughness that produces the no-slip behavior (and thus $f \neq 0$) on the molecular level, even when the roughness is considerably less than the viscous sublayer thickness. Such pipes are called *hydraulically smooth*. Various investigators have attempted to obtain an analytical expression for $f = \varphi(Re, \varepsilon/D)$. Note that the Moody chart covers an extremely wide range in flow parameters. The non-laminar region covers more than four orders of magnitude in Reynolds number from $Re = 4 \cdot 10^3$ to $Re = 10^8$. Obviously, for a given pipe and fluid, typical values of the average velocity do not cover this range. However, because of the large variety in pipes D, fluids (ρ, and μ and and velocities (v), such a wide range in Re is needed to accommodate nearly all applications of pipe flow.

Colebrook combined all data for transition and turbulent flow in smooth as well as rough pipes into the following relation known as **Colebrook equation**

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}}\right) \tag{6}$$

The Colebrook equation is implicit in f, and determination of friction factor requires tedious iteration. An approximate explicit relation for f is given by S.E. Haaland in 1983 as

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \tag{7}$$
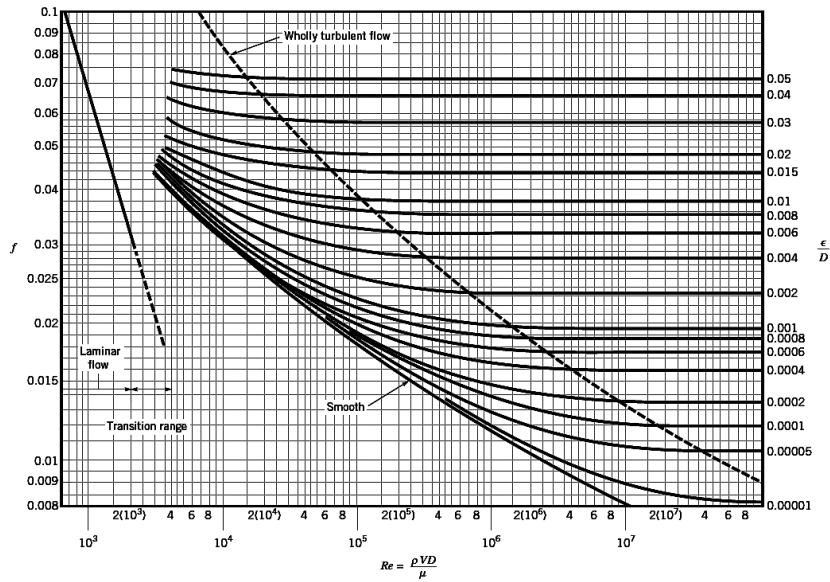
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Figure 3: Friction factor as a function of Reynolds number and relative roughness for round pipes
-the **Moody chart**

For hydraulically smooth pipe the friction factor is approximated by Blasius (1911) formula

$$f = (100 \, Re)^{-1/4} \tag{8}$$

The next formula proposed by Aldsul(1952) gained some popularity in the engineering application due to its simplicity:

$$f = 0,11(\frac{\varepsilon}{D} + \frac{68}{Re})^{1/4} \tag{9}$$

It is clear that in order to use the Moody diagram we must be able to obtain values of surface roughness. These have been measured and tabulated (and sometimes plotted) for an extensive range of materials used in piping systems. Table 1 provides some representative values.

Table 1. Surface roughness values for various engineering materials

PIPING MATERIAL	ROUGHNESS ε *mm*
Cast iron	0.26
Commercial steel and wrought iron	0.045
Concrete	0.3-3.0
Drawn tubing	0.0015
Galvanized iron	0.15
Plastic,(and glass)	0.0 (smooth)
Riveted steel	0.9-9.0

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We must remember that the values in table typically used, are not actual measured ones, but are instead the result of data correlations constructed over a range of measurements. They are sometimes referred to as "equivalent" roughnesses; it is useful to consider them as simply representative values.

1.2 Types of Fluid Flow Problems

In the design and analysis of piping systems that involve the use of the Moody chart, we usually encounter three types of problems:

1. Determining the **pressure drop** when the the pipe length and diameter are given for a specified flow rate (or velocity)

2. Determining the **flow rate** when the the pipe length and diameter are given for a specified pressure drop

3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop.

Example 1. *Oil, with $\rho = 900kg/m3$ and kinematic coefficient of viscosity $\nu = 0,00001m2/s$, flows at $q_v = 0,2m3/s$ through 500 m of 200-mm diameter cast-iron pipe. Determine (a) the head loss and (b) the pressure drop if the pipe slopes down at 10 in the flow direction.*
Solution. *First we compute the Reynolds number $Re = V \cdot d/\nu = 4 \cdot q_v/(\pi d \nu) = 4 \cdot 0,2/(3,14 \cdot 0,2 \cdot 0,00001) = 128000$. Velocity is equal to $v = q_v/(\frac{\pi d^2}{4}) = 6.4$ m/s. Absolute roughness for iron-cast pipe is $\varepsilon = 0.26$ mm. So relative roughness is $\varepsilon = \frac{\varepsilon}{d} = 0,0013$. Now we are able to calculate using the formula(10) or Moody chart, friction factor $f = 0,0225$. Then the head loss is*

$$h_f = f\frac{L}{d}\frac{v^2}{2g} = 0,0225\frac{500}{0.2}\frac{6,4^2}{2 \cdot 9,81} = 117 \ m$$

For inclined pipe the head loss is

$$h_f = \frac{\Delta p}{\rho g} + z_1 - z_2 = \frac{\Delta p}{\rho g} + L\sin 10^o.$$

So pressure drop is

$$\Delta p = \rho g(h_f - 500 \cdot \sin 10^o) = 900 \cdot 9,81 \cdot (117 - 87) = 265 \cdot 10^3.$$

Example 2. *Oil, with $\rho = 950 \ kg/m3$ and $\neq= 2E-5 \ m2/s$, flows through a $30-cm$-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio is $\varepsilon/d = 0.0002$. Find the average velocity and flow rate.*
Iterative Solution. *To start we need to guess f. A good first guess is the "fully rough" value (wholly turbulent) for $\varepsilon/d = 0.0002$ from Moody chart. It is $f \approx 0.015$. Now from Darcy-Weisbach formula (4) we have*

$$h_f = f\frac{L}{D}\frac{v^2}{2g} \qquad \Rightarrow \qquad fv^2 = 0.471$$

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$$f \approx 0.015 \qquad v = \sqrt{0.471/0.014} = 5.8 \ m/s \qquad Re = vd/v \approx 87000$$
$$f_{new}(87000) = 0.0195 \quad v = \sqrt{0.471/0.0195} = 4.91 \ m/s \qquad Re = vd/v \approx 73700$$
$$f_{new}(73700) = 0.0201 \quad v = \sqrt{0.471/0.0201} = 4.84 \ m/s \qquad Re = vd/v \approx 72600$$

$f(72600) = 0.0201$, *so we can accept* $v = 4,84 \ m/s$, $\quad q_v = v(\frac{\pi d^2}{4}) = 0,342 \ m^3/s$.

1.3 Minor losses

For any pipe system, in addition to the Moody-type friction loss computed for the length of pipe. Most pipe systems consist of considerably more than straight pipes. These additional components add to the overall head loss of the system. Such losses are generally termed **minor losses**, with the apparent implication being that the majority of the system loss is associated with the friction in the straight portions of the pipes, the **major losses** or **local losses**. In many cases this is true. In other cases the minor losses are greater than the major losses. The minor losses may raised by

1. Pipe entrance or exit

2. Sudden expansion or contraction

3. Bends, elbows, tees, and other fittings

4. Valves,open or partially closed

5. Gradual expansions or contractions

The **major losses** may not be so minor; e.g., a partially closed valve can cause a greater pressure drop than a long pipe. The losses are commonly measured experimentally. The data, especially for valves, are somewhat dependent upon the particular manufacturers design.
The most common method used to determine these head losses or pressure drops

$$\boxed{h_L = \frac{\Delta p}{\rho g} = K_L \frac{v^2}{2g}} \tag{10}$$

where K_L means **(local) loss coefficient**. Although K_L is dimensionless, it is not correlated in the literature with the Reynolds number and roughness ratio but rather simply with the raw size of the pipe. Almost all data are reported for turbulent-flow conditions.

Example 3. *Find (see figure 5.)*

1. *the discharge through the pipeline as in figure for H=10 m*

2. *determine the head loss H_1 for $q_v = 60 \ l/s$.*

$D = 0.15m$, $\frac{\varepsilon}{D} = 0,0017$, $v = 1,01 \cdot 10^{-6}$.
Solution. *The energy equation applied between points 1 and 2 including all the losses, can be written as*

$$H_1 = \frac{v^2}{2g} + f \frac{L}{D} \frac{v^2}{2g} + (K_1 + K_2 + K_3 + K_4) \frac{v^2}{2g}$$

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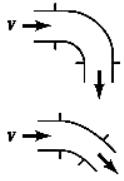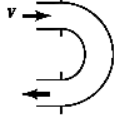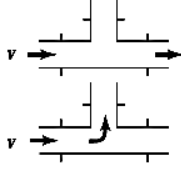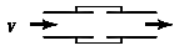**Loss Coefficients for Pipe Components** $\left(h_L = K_L \dfrac{V^2}{2g} \right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

Figure 4:

where $K_1 = 0,5$ is entrance loss coefficient, $K_2, K_3 = 0,9$ as a standard elbow, and $K_4 = 10$ for globe valve fully open. Putting the values tot the above formula one obtain

$$H_1 = \frac{v^2}{2g}(13,3 + 680\ f)$$

To start we need to guess f. A good first guess is the "fully rough" value (wholly turbulent) for $\varepsilon/d = 0,0017$ from Moody chart. It is $f \approx 0,022$. Now from Darcy-Weisbach formula (4) we have

$$10 = \frac{v^2}{2g}(13,3 + 680\ 0,022) \qquad \Rightarrow \qquad v_2 = 2.63, \quad Re = 391000$$

$f \approx 0,023$ $v = 2,6\ m/s$ $Re = vd/v \approx 380000$

$f_{new}(380000) = 0,023$ $v = 2,6\ m/s$ $Re = vd/v \approx 380000$
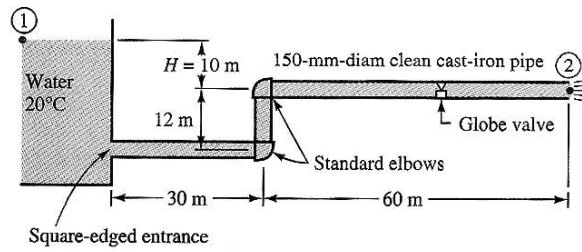
Figure 5: Pipeline with minor losses

We accepted $v = 2,60\ m/s$, $q_v = v(\frac{\pi d^2}{4}) = 45,9 \cdot 10^{-3}\ m^3/s$.
For the second part, with q_v known, the solution is straightforward:

$$v_2 = \frac{q_v}{A} = \frac{0.06}{\frac{\pi 0.15^2}{4}} = 3.40\ m/s, \qquad Re = 505000, \qquad f = 0.023$$

and

$$H_1 = \frac{3,4^2}{2 \cdot 9,81}(13,3 + 680\ 0,023) = 17,06\ m$$

Example 4. *Flow between two reservoirs. Water at 10^oC flow from a large reservoir to a small one through a 5-cm-diameter cast iron piping system shown in figure (6). Determine the elevation z_1 for the flow rate of $6l/s$.*
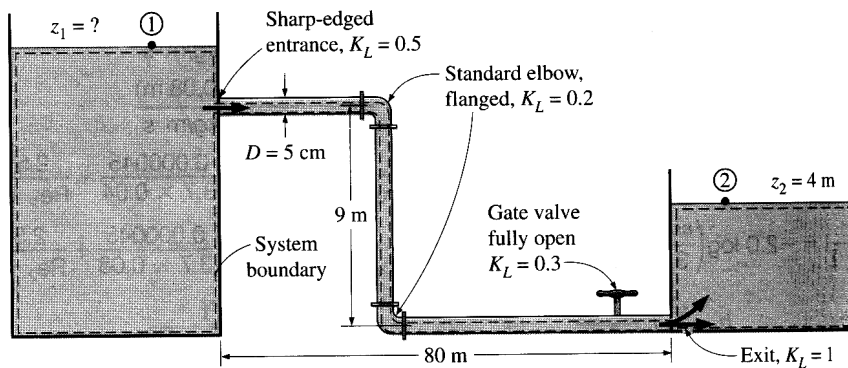

Figure 6:

(Answer. 27,9 m)

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During this course I will be used the following books:

References

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